8. I. E. Morichev, A. P. Onokhov, and V. P. Savinov, "The temperature dependence of the absorption coefficient and optical breakdown of glass," Zh. Tekh. Fiz., 49, No. 12, 2652-2655 (1979).
9. L. D. Landau and E. M. Lifshits, Elasticity Theory, Pergamon Press (1970).

TEMPERATURE DEPENDENCE OF THE ELECTRICAL RESISTIVITY AND NORMAL INTEGRAL COEFFICIENT OF RADIATION OF METALS

L. N. Aksyutov and E. N. Maleev

UDC 537.311.3:536.45

On the basis of an analysis of the values of the normal integral coefficient of radiation (radiative capacity) and dc resistivity of metals, we obtain formulas describing the temperature dependence of these quantities and their interrelationship over a wide range of temperatures.

A great deal of information has already been accumulated concerning the temperature dependence of the electrical and radiative properties of metals. However, in a number of cases the experimental data found in different literature sources are not in good agreement. Consequently, the available information must be generalized in a way that will make it possible to establish the degree of reliability required for the correct solution of practical problems and will provide a method for its adequate theoretical interpretation.

The purpose of the present article is to determine — on the basis of a generalization of the results of an experimental investigation of the electrical resistivity (ρ_μ) and the integral normal coefficient of radiation (ε_{tn}) of metals — the characteristics of the temperature-dependent variation of these quantities at temperatures below the melting point.

We carried out an analysis of the experimental data for ε_{tn} and ρ_μ taken from various literature sources [1-5, 12-14], as well as the values of ε_{tn} calculated from the results of an investigation of the optical constants of metals [6] and from the integral hemispherical radiative capacity (ε_{th}) by means of the formulas [7]:

$$\frac{\varepsilon_{tn}}{\varepsilon_{th}} = 1 + \exp[-(5.2\varepsilon_{tn} + 1.16)],\ \varepsilon_{tn} < 0.2; \quad (1)$$

$$\frac{\varepsilon_{tn}}{\varepsilon_{th}} = 7.37 \cdot 10^{-2} + \exp[-(0.11 \ln \varepsilon_{tn} + 0.14)],\ \varepsilon_{tn} \geqslant 0.2. \quad (2)$$

In Fig. 1, where the values of ρ_μ and ε_{tn} for a number of the metals we considered are plotted against temperature, it can be seen that a change in the temperature dependence of these quantities in some metals, e.g., in nickel (see Fig. 1c), takes place at the same temperature. In other metals, in particular in tungsten (Fig. 1a), the relation $\varepsilon_{tn} = f(T)$ changes at a temperature that lies in the interval 1800-1900°K, while the function $\rho_\mu = \varphi(T)$ remains unchanged over the entire temperature range represented in the figure. This fact contradicts the conclusion drawn in [8] to the effect that the variation of $\rho_\mu(T)$ for metals as the temperature varies is similar to that of $\varepsilon_{tn}(T)$.

For most metals the experimental values of ρ_μ and ε_{tn} were obtained at temperatures lower than the melting point T*, which, as is known, is a characteristic quantity for each specific metal. Obviously, ρ_μ^* and ε_{tn}^*, corresponding to the solid phase of the metal at the melting point, can be characteristic quantities of the same kind.

Since the temperature dependence of the electrical resistivity of metals, as of all crystalline solids, remains unchanged in the range of high temperatures up to the melting point [9], the numerical value of ρ_μ^* for each metal can be determined from the high-temperature segment of curves of the type represented in Fig. 1 by extrapolating them to T = T*. The numerical value of ε_{tn}^* is determined in the same way.

We found that the electrical resistivity, referred to the appropriate absolute temperature and expressed in terms of its characteristic quantity and the melting point, is a power function of the relative temperature:

Translated from Inzhenero-Fizicheskii Zhurnal, Vol. 45, No. 1, pp. 132-139, July, 1983. Original article submitted February 15, 1982.

0022-0841/83/4501-0817$07.50

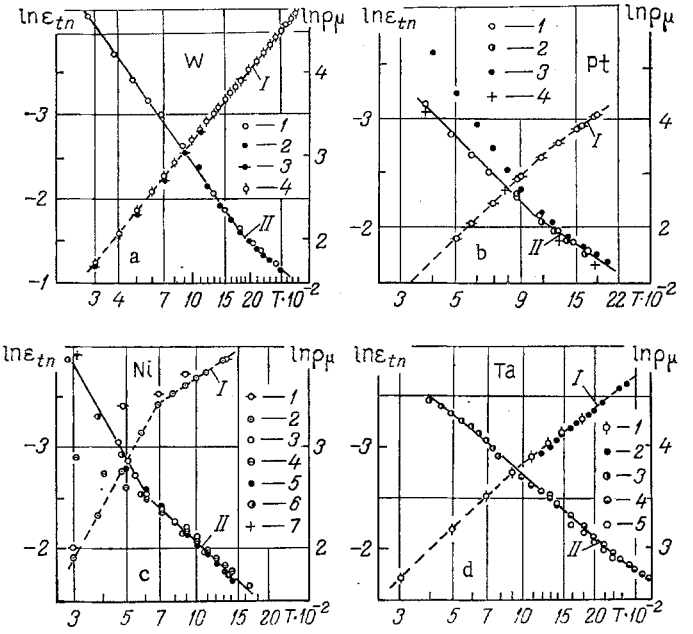

Fig. 1. Temperature dependence of the electrical resistivity (I) and the normal integral coefficient of radiation (II) of metals: a) tungsten: ρ_μ — 3 [12], 4 [5]; ε_{tn} — 1 [6], 2 [3]; b) platinum: ρ_μ — data of [2] and [4]; ε_{tn} — 1 [6], 2 [13], 3 [3], 4 [2]; c) nickel: ρ_μ — 1 [2], 2 [5]; ε_{tn} — 3 [11], 4 [3] (the data of 3, 4 are converted by formulas (1), (2) from the values of ε_{th} given in the studies indicated), 5 [3], 6 — our data, 7 [1]; d) tantalum: ρ_μ — 1 [2], 2 [12]; ε_{tn} — 3 [14], 4 [3] — conversion of the values of ε_{th}, 5 [3]; T, °K.

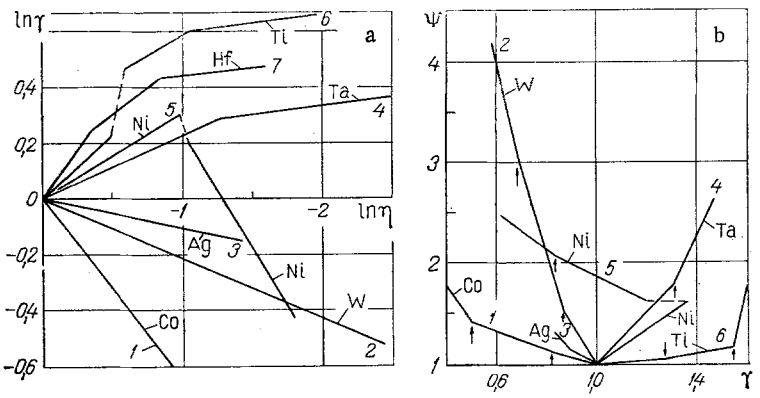

Fig. 2. Graphs of the functions represented by Eqs. (3) (a) and (5) (b) for Co (1), W (2), Ag (3), Ta (4), Ni (5), Ti (6), and Hf (7) (the data for Hf are not shown in Fig. 2b).

$$\gamma = a(\eta)^b, \tag{3}$$

where $\eta = T/T^*$; $\gamma = (\rho_\mu/T)/(\rho_\mu^*/T^*)$; a and b are coefficients whose numerical values are different for each metal.

From Eq. (3) we obtain an expression for the electrical resistivity

$$\rho_\mu = \rho_\mu^* a \left(\frac{T}{T^*}\right)^{1+b}, \tag{4}$$

which differs radically from the previously known formulas.

TABLE 1. Values of T*, ε^*_{tn}, ρ^*_μ, and the Coefficients α, b, c, and d from Formulas (4) and (6)

Metal	T^*, K [2]	ε^*_{tn}	ρ^*_μ, $\mu\Omega\cdot$cm	a	b	c	d	Temperature range, °K
Cu	1357	0,0291	9,2	1	0,112	—2,03	3,03	$T\geqslant 580$
						—1,5	2,55	$T<580$
Ag	1235	0,0245	7,7	1	0,1	—1,48	2,48	$T\geqslant 480$
						—2,68	3,58	$T<480$
Au	1337	0,1022	12,5	1	0,118	—5,12	6,12	$T\geqslant 450$
						—8,71	9,31	$T<450$
Be	1560	0,390	56,4	1	0,28	—10,57	11,57	$T\geqslant 640$
						—6,99	8,77	$T<640$
Al	933,5	0,0376	10,8	1	0,153	—1,90	2,90	$T\geqslant 436$
						—2,20	3,17	$T<436$
Mo	2900	0,285	81,4	1	0,15	—5,77	6,76	$T\geqslant 1583$
						—7,96	8,76	$640\leqslant T<1583$
						—10,33	10,65	$T<640$
W	3660	0,365	117,2	1	0,21	—3,85	4,85	$T\geqslant 1878$
						—8,21	8,64	$683\leqslant T<1878$
						—10,5	10,25	$T<683$
Co	1767	0,219	135,0	1	0,65	—0,69	1,69	$T\geqslant 1174$
						—0,99	1,92	$697\leqslant T<1174$
						—2,71	2,86	$T<697$
Rh	2236	0,1625	51,7	1	0,183	—2,0	3,0	$T\geqslant 1413$
						—4,73	5,51	$T<1413$
Ti	1940	0,312	195,5	1	—0,450	1,136	—0,136	$T\geqslant 1200$
				1,347	—0,296	0,57	0,432	$700\leqslant T<1200$
				1,71	—0,06	2,43	—2,95	$T<700$
Zr	2128	0,252	164,7	1	—0,265	0,675	0,325	$T\geqslant 1200$
				1,116	—0,522	0,203	0,809	$700\leqslant T<1100$
				1,86	—0,06	7,86	—14,16	$T<700$
Hf	2222	0,325	195,5	1	—0,711	0,285	0,715	$T\geqslant 1570$
				1,113	—0,387	0,405	0,565	$960\leqslant T<1570$
				1,47	—0,06	14,75	—21,55	$T<960$
V	2190	0,250	116,0	1	—0,243	2,15	—1,15	$T\geqslant 1027$
				1,14	—0,07	9,88	—10,39	$T<1027$
Nb	2742	0,245	87,4	1	—0,298	1,83	—0,83	$T\geqslant 910$
				1,32	—0,07	4,16	—4,27	$T<910$
Ta	3270	0,294	113,0	1	—0,233	2,58	—1,58	$T\geqslant 1600$
				1,23	—0,07	7,39	—8,04	$T<1600$
Ni	1728	0,210	59,7	1	—0,308	1,66	—0,66	$T\geqslant 630$
				2,94	0,85	—1,167	3,03	$395\leqslant T<630$
						1,847	3,61	$T<395$
Pt	2045	0,192	63,0	1	—0,219	1,82	—0,82	$T\geqslant 800$
				1,13	—0,028	18,3	—19,8	$T<800$

The value of the integral normal coefficient of radiation for T < T* according to the results of our analysis can be expressed by the following empirical relation:

$$\psi = \frac{\ln \varepsilon_{tn}}{\ln \varepsilon^*_{tn}} = c\gamma + d, \tag{5}$$

from which we obtain

$$\varepsilon_{tn} = (\varepsilon^*_{tn})^{c\gamma+d}. \quad (6)$$

Thus, the numerical value of the integral normal coefficient of radiation of metals is expressed not in terms of the absolute values of their electrical resistivity and temperature, as is the case in the well-known Foot formula or its modifications [10], but in terms of a characteristic value of this coefficient and the relative temperature dependence γ of the reduced electrical resistivity.

Table 1 contains the values of the quantities occurring in formulas (4) and (6), by means of which it is possible to calculate ρ_μ and ε_{tn} for the tabulated metals at temperatures below the melting point.

To illustrate the results of our analysis, in Fig. 2 we show the graphs on the basis of which we obtained the expressions (3) and (5). The arrows in Fig. 2b indicate the temperature intervals in which the values of the coefficients c and d are constant for the metal in question.

Our analysis enabled us to establish the following features of the temperature dependence of ρ_μ and ε_{tn}.

1. All the metals we considered formed two groups, which differed from each other in the sign of the coefficient b in formula (3).

The group with positive coefficient b (the P group) included Ag, Cu, Au, Mo, Al, Rh, W, Be, and Co, for which the function γ was unchanged throughout the temperature range considered. The second group (the N group), characterized by negative values of b, consisted of metals such as Ti, Zr, Hf, V, Nb, Ta, Ni, Pd, and Pt. The characteristic feature of this group is the presence of two or three temperature intervals in which there is a change in the function γ (see Table 1 and Fig. 2a). The high-temperature boundary of the intervals is close to the temperature value (T_{st}) at which there is a structural change in the metal under consideration. From the data of [5], this temperature for titanium (T_{st} = 1155 ± 20°K) and hafnium (T_{st} = 1583 ± 10°K) corresponds to the transition from a hexagonal lattice structure to a body-centered cubic structure, for tantalum (T_{st} = 1573°K) it corresponds to the recrystallization temperature, and for nickel (T_{st} = 638°K) it corresponds to the point of magnetic transformation, known as the Curie point, at which we observe not only an anomalous change in the radiative capacity [11], but also the completely obvious fact (see Fig. 2) that nickel undergoes a transition from the N group of metals to the P group.

The results we obtained did not support the universality of the classification according to which the electrical resistance of metals is related to their structure-insensitive properties [15].

2. The additional changes in the temperature characteristic ρ_μ of titanium, zirconium, and hafnium for $T < T_{st}$ apparently are of the same nature, as demonstrated by the equality of the coefficients b for these metals when T < 700°K. The low-temperature segments of the function γ for other metals of the N group are also characterized by nearly equal values of the coefficient b (see V, Nb, and Ta in Table 1).

On the assumption that the temperature characteristic of the function γ for metals does not change when T < 300°K, we calculated from the data of Table 1 the values of ρ_μ for low temperatures, which are shown in Table 2.

The agreement between the calculated and experimental values must be regarded as satisfactory if we take account of the possible experimental error resulting from errors in reading the numerical values of ρ_μ for all metals except zirconium from the graphs of [16]. On the basis of this, we can conclude that the value of the electrical resistivity of metals for T > 50°K can be estimated from the relation $\rho_\mu = \varphi(T)$ measured in the temperature range 300-700°K.

3. The temperature characteristic ε_{tn}, regardless of whether the metal belongs to the P group or the N group, has two or three segments for which the values of the coefficients c and d are different. Only for some metals of the N group do these segments coincide with the temperature intervals in which the function γ changes.

The lack of an unambiguous connection between the temperature characteristics ρ_μ and ε_{tn} indicates a difference between the physical mechanisms determining the values of these quantities in different regions of the temperature range considered in the present study. This difference is particularly evident for metals of the P group.

TABLE 2. Low-Temperature Values of the Electrical Resistivity of Some Transition Metals, Calculated from the Data of Table 1, and the Experimental Data from [16]

Metal	Method	ρ_μ, μΩ·cm					
		Temperature, °K					
		50	100	150	200	250	300
W	I	0,6	1,5	2,4	3,5	4,6	5,7
	II	0,5	1,2	2,3	3,5	4,5	5,5
Mo	I	0,8	1,7	2,7	3,8	4,9	6,0
	II	0,6	1,5	2,6	4,0	5,0	6,2
Nb	I	2,8	5,3	7,7	10,1	12,4	14,7
	II	1,5	4,5	7,5	10,0	12,5	14,5
Ta	I	2,8	5,4	7,9	10,3	12,7	15,1
	II	1,5	4,5	6,5	9,0	11,5	14,0
Zr	I	9,0	17,3	25,3	33,2	40,9	48,6
	II	4,0	9,5	19,0	28,0	36,0	43,5
Hf	I	8,1	15,6	22,8	29,9	36,9	43,8
	II	9,2	15,5	22,0	28,0	34,0	39,5

Note: I) calculated values; II) experimental values.

TABLE 3. Calculation of the Values of ρ_μ, ε_{tn}, and ε_{th} for Copper, Aluminum, and Silver on the Basis of the Data of Table 1, and Results of the Measurement of ε_{th} for Low (T < 300°K) [17] and High (T > 300°K) [18] Temperatures

T, K	ρ_μ, μΩ·cm			ε_{tn}, %			ε_{th}, %					
							Cu		Al		Ag	
	Cu	Al	Ag	Cu	Al	Ag	I	II	I	II	I	II
140	0,74	1,21	0,70	0,74	0,67	0,49	0,96	0,42	0,87	—	0,64	—
160	0,85	1,41	0,81	0,77	0,74	0,55	1,00	0,78	0,96	—	0,72	—
180	0,97	1,62	0,92	0,82	0,83	0,59	1,07	0,97	1,08	1,03	0,77	—
200	1,09	1,83	1,04	0,86	0,92	0,68	1,12	1,18	1,19	1,22	0,88	—
250	1,40	2,36	1,33	0,97	1,10	0,83	1,26	1,27	1,43	1,58	1,08	—
290	1,65	2,81	1,56	1,04	1,28	0,91	1,35	1,41	1,82	1,77	1,18	—
400	2,36	4,07	2,23	1,22	1,74	1,24	1,58	1,56	2,24	2,23	1,60	1,58
500	3,03	5,26	2,85	1,39	2,14	1,53	1,79	1,77	2,74	2,70	1,97	1,90
600	3,71	6,49	3,48	1,55	2,51	1,67	2,00	2,04	3,20	3,18	2,15	2,15
700	4,41	7,75	4,12	1,75	2,87	1,80	2,25	2,26	3,64	3,65	2,31	2,33
800	5,11	9,04	4,77	1,92	3,25	1,93	2,46	2,41	4,11	—	2,48	2,50
900	5,82	10,36	5,44	2,09	3,64	2,07	2,68	2,68	4,58	—	2,65	2,63

Note: I) calculated values; II) experimental values.

TABLE 4. Low-Temperature Values of ε_{th}, %, for Tungsten

T, K	Formula		Calc. from Table 1
	(7)	(8)	
100	0,94	0,47	0,61
150	1,34	0,78	0,95
200	1,64	1,11	1,33
250	1,96	1,47	1,74
300	2,27	1,84	2,19
400	2,86	2,62	3,21
500	3,42	3,45	4,37
600	4,46	4,32	5,67
700	5,66	5,46	7,05
800	6,97	6,77	8,24
900	8,36	8,19	9,48

The reliability of the relation shown in Table 1 between the functions $\varepsilon_{tn} = f(T)$ and $\rho_\mu = \varphi(T)$ is confirmed by a comparison of the calculated and experimental values of ε_{th} shown for silver, copper, and aluminum in Table 3. The calculation was carried out on the basis of the data of Table 1 by using formulas (3)-(5) and (1).

It can be seen from Table 3 that for $T \geqslant 180°K$ the divergence between the calculated and experimental values of ε_{th} does not exceed 10%, which corresponds to the experimental error of [17]. For lower temperatures the divergence becomes substantial and cannot be adequately interpreted because of the great variety of factors that may influence the quantity ε_{th} being measured. Significant results in this respect were obtained in the measurements of ε_{th} made by identical methods for polycrystalline [19] and monocrystalline [20] specimens of high-purity tungsten. The maximum error of the experimental data is estimated by the authors of these studies to be 8% and 20%, respectively. For convenience of comparison, we approximated the aforementioned experimental data by the following expressions, with an error not exceeding 1%. For the data of [19]:

$$\varepsilon_{th} = \exp(0.802 \ln T - 3.754), \quad T \leqslant 512\ \text{K};$$

$$\varepsilon_{th} = \exp(1.55 \ln T - 8.42), \quad 512\ \text{K} < T \leqslant 1000\ \text{K} \tag{7}$$

and for the data of [20]:

$$\varepsilon_{th} = \exp(1.234 \ln T - 6.43), \quad T \leqslant 623\ \text{K};$$

$$\varepsilon_{th} = \exp(1.61 \ln T - 8.85), \quad 623\ \text{K} < T \leqslant 1254\ \text{K}. \tag{8}$$

Table 4 shows the values of ε_{th} for tungsten calculated by formulas (7) and (8) and also from the data of Table 1. While for $T > 600°K$, as can be seen from the table, the divergence between the experimental results of [19] and [20] does not exceed 4%, we see that for low temperatures it increases, amounting to about 25% for $T = 300°K$ and increasing as the temperature decreases further. Such a divergence between the ε_{th} values should probably be attributed to the difference between the crystal structures of the specimens investigated. Our own data shown in Table 4 were obtained for technically pure polycrystalline metal with a polished surface.

A comparison of calculated and experimental values, according to the results shown in Tables 2-4, enables us to conclude that the data of Table 1 can be used for engineering calculations in solving problems in which it must be known how the electrical and integral radiative properties of metals vary as functions of temperature over a range from cryogenic temperatures to the melting point. Further investigations of these properties of metals will make it possible not only to refine the accuracy of the relations and values found in the present study, but also to give a theoretical explanation for them.

NOTATION

ε_{tn}, ε_{th}, integral normal and hemispherical coefficients of radiation; ρ_μ, electrical resistivity, $\mu\Omega \cdot cm$; T, absolute temperature, °K; T*, melting point, °K.

LITERATURE CITED

1. L. A. Novitskii and B. M. Stepanov, Optical Properties of Materials at Low Temperatures (Handbook) [in Russian], Mashinostroenie, Moscow (1980).
2. G. V. Samsonov (ed.), Properties of the Elements (Handbook) [in Russian], Part 1, Metallurgiya, Moscow (1976).
3. A. E. Sheindlin (ed.), Radiative Properties of Solids (Handbook) [in Russian], Énergiya, Moscow (1974).
4. V. A. Golovin and É. Kh. Ul'yanova, Properties of Noble Metals and Alloys (Handbook) [in Russian], Metallurgiya, Moscow (1964).
5. W. Espe, Materials of High Vacuum Technology [Russian translation], Vol. 1, GÉI, Moscow–Leningrad (1962).
6. L. N. Aksyutov, "Integral radiative properties of gold, platinum, and tungsten," Inzh.-Fiz. Zh., 28, No. 5, 826-828 (1975).
7. L. N. Aksyutov, "On the problem of determining the hemispherical radiative capacity of metals," Inzh.-Fiz. Zh., 40, No. 5, 927-928 (1981).
8. A. J. Sievers, "Thermal radiation from metal surfaces," JOSA, 68, No. 11, 1505-1516 (1978).
9. A. Ubbelohde, Melting and Crystal Structure [Russian translation], Mir, Moscow (1969).
10. V. É. Peletskii, "The connection between the radiative capacity and the statistical conductivity of metals," in: Thermophysical Properties of Materials at High Temperatures [in Russian], High-Temperature Inst., Academy of Sciences of the USSR, Moscow (1978), pp. 95-104.

11. G. L. Zuppardo and K. G. Ramanathan, "Anomalous emissivities of nickel and iron near their Curie temperatures," JOSA, 61, No. 12, 1607-1612 (1971).
12. Ya. A. Kraftmakher, "Specific heat at high temperatures and formation of vacancies in high-melting metals," in: Investigations at High Temperatures [in Russian], Nauka, Novosibirsk (1966), pp. 5-54.
13. R. E. Rolling, "Effect of surface roughness on the spectral and total emittance of platinum," AIAA Paper No. 320 (1967).
14. R. Blickensverfer, D. K. Deadorf, and R. L. Lincoln, "Normal (spectral) total emittance at 400-850°K and normal spectral reflectance at room temperature of Be, Hf, Ta, Nb, Ti, V, and Zr," J. Less-Common Met., 51, No. 1, 13-23 (1977).
15. E. M. Savitskii and G. S. Burkhanov, Rare Metals and Alloys. Physicochemical Analysis and Metallurgy [in Russian], Nauka, Moscow (1980).
16. F. W. Clinard and C. P. Kempter, "Low-temperature properties of some transition metals and transition-metal carbides," J. Less-Common Metals, 15, No. 2, 59-72 (1968).
17. E. A. Estalote and K. G. Ramanathan, "Low-temperature emissivities of copper and aluminum," JOSA, 67, No. 1, 39-44 (1977).
18. K. G. Ramanathan and S. H. Yen, "High-temperature emissivities of copper, aluminum and silver," JOSA, 67, No. 1, 32-38 (1977).
19. D. P. Verret and K. G. Ramanathan, "Total hemispherical emissivity of tungsten," JOSA, 68, No. 9, 1167-1172 (1978).
20. L. A. Wojcik, A. J. Sievers, G. W. Gracham, et al., "Total hemispherical emissivity of W (100)," JOSA, 70, No. 4, 443-450 (1980).